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CONCRETE GEOMETRY IN THE JUNIOR HIGH SCHOOL

W. H. FLETCHER

Director of Course for Training Junior-High-School Teachers
State Normal School, Oshkosh, Wisconsin

One of the results which will naturally follow the movement to revise the curriculum of the secondary school will be that such studies as mathematics and foreign languages will be thrown into the elective group. It will no longer be required for graduation from a secondary school that a pupil shall have taken geometry of the demonstration type, ancient or modern foreign languages, and possibly other courses. This is inevitable if the various demands for economy in learning, especially in the last three years of the secondary school, are realized and fulfilled.

Rugg and Clark, in their monograph entitled *Scientific Method in the Reconstruction of Ninth-Grade Mathematics*, make this significant statement:

The point of view that will be taken throughout this discussion is that ninth-grade mathematics will certainly come to be regarded by schoolmen in our generation as the last year of mathematics that will be required. There are marked evidences of the tendency to lead mathematical instruction through a reorganized intermediate school mathematics into a last-year required course in the ninth grade.

Another result from such a revision of the curriculum will be that much of great cultural value will be lost to the student who, through inadequate guidance, fails to choose such courses as those mentioned. Those students who do not study the mathematics of the secondary school, as now provided, will not get a very adequate notion of space relations and their importance to our present civilization. Their outlook upon

life will be just so much narrower. Their appreciation of the value to mankind of the simplest applied mathematics will be vastly less than it should be to enable them to understand that phase of human activity.

Since the beginning of the secondary-school period is now considered to be properly placed at the seventh grade, still other problems have arisen. Prominent among these is the question of what shall be the mathematics in the seventh and eighth grades, or, as they are generally called, the first two years of the junior high school. It is important, therefore, to provide a good course in mathematics for the seventh, eighth, and ninth grades, without a great amount of repetition and a consequent lack of interest on the part of the pupils.

Now the essential aim of the junior high school is to give pupils of these grades a broad general view of life. The state of development of pupils in these years demands such an aim, and the fact that in the senior high school they must begin to specialize makes this aim doubly imperative. During the years of the junior high school the various curricula should furnish subject-matter that will show to the pupils the modern world in all its most important aspects. In other words, the experience of the pupil must reach out into other fields than those which he has explored during the first six years of his course.

Concrete geometry aims to furnish the student with a definite set of notions regarding space relations, with reference to those applications of geometry which are a part of his immediate environment. By means of measurement, construction, estimating, designing, observation, and appreciation the field of applied geometry is studied. A great deal of manipulation of the truths of geometry is secured in this way. But the aim is primarily to teach the subject by so presenting it that the pupils are enabled to sense the various facts, thus becoming thoroughly familiar with the various relations of

magnitudes and understanding the importance of space relations in their environment. This is another attempt to apply that generally accepted principle of learning by doing. It is also an excellent example of the belief that art should precede science in all studies of the curriculum, especially during the junior-high-school period.

A course in concrete geometry to satisfy such aims and demands is taught in the seventh grade in the training school of the State Normal School at Oshkosh. The purpose of this article is to outline the content and method of presenting this subject to children of the seventh school year. It is evident that the subject-matter of such a course must have a definite appeal to children of this age. It must be really concrete and must be studied from the standpoint of the pupil and his power to understand.

The subject-matter consists of not more than one-fourth of the propositions of Euclid as found in any standard high-school text on this subject. One of the foremost writers on the teaching of geometry¹ is authority for the statement that "Not more than twenty-five per cent of propositions have any genuine application outside of Geometry." Such propositions as the following are evidently the ones upon which we base a great deal of our measurement, construction, and design: most of the propositions having to do with triangles; all of the construction propositions, such as the bisection of an angle, the construction of an angle equal to a given angle, constructing circles in triangles and around triangles, drawing circles through three points not in a straight line, tangents to circles; similarity of triangles; all propositions dealing with areas; construction of all kinds of figures; problems dealing with symmetry; and a few of the propositions in solid geometry dealing with volumes and surfaces of solids. Such subject-matter as this furnishes a basis only. Any attempt to prove

¹ DAVID EUGENE SMITH, *The Teaching of Geometry*, p. 74

these propositions by logical proof on the part of children of the seventh grade would be impossible. For this reason, these propositions must be elaborated into a series of concrete problems which are within the experience of children of the seventh grade, and so chosen that the children can solve them with increasing ability, and of such a nature that they can gain from their study the results which have been named as the aim in the study of concrete geometry.

Practically every book on plane geometry which has been published since the year 1910 has made considerable use of the type of problem which we have just mentioned. These problems consist of designs, applications of subject-matter of plane geometry, and a little graphical representation of geometrical truths, but the quantity of this material has been so greatly subordinated to the standard propositions that any attempt to use such texts as a basis for a course in concrete geometry would be impossible. Certain types of work which are vitally necessary in any seventh-grade course are entirely lacking.

The course now being taught at Oshkosh makes a great deal of use of problems involving the measurement of rooms, grounds, walks, and all kinds of rectilinear areas to accustom children to the importance of measurement. In the same way there are introduced the measurement of angles, the construction of areas, angles, and figures of all kinds as found in buildings and construction work of all sorts, designs of wallpaper, linoleums, flooring, and forms of architecture where designs are applied.

The subject-matter is divided into three groups: first, those types of problems which are used to teach the use of any particular kind of instrument, such as the pupils need to use in order to do quick and accurate work; second, those problems which the teacher introduces for the sake of teaching new geometrical truths, together with sufficient additional problems along the same general line to make sure that such a geometri-

cal truth becomes fixed; third, those problems which the children originate and bring into class for solution by their classmates. In the early part of the course the problems of the first and second groups predominate in number, but as the course proceeds the problems suggested by the class become so numerous that pupils furnish a large part of their own problems for solution and those almost invariably of a difficulty entirely sufficient.

The following list includes nearly all of the truths of geometry taught in the seventh grade:

1. Meaning represented by such terms as horizontal, vertical, perpendicular, oblique, bisect, trisect, locus of a point.
2. Study of angles—right, acute, obtuse—also their unit of measurement.
3. Study of triangles—right, scalene, isosceles, equilateral, obtuse.
4. Study of quadrilaterals, such as rectangle, parallelogram, square, rhombus, rhomboid, trapezoid, including all facts about each one.
5. Regular polygons—all the facts, such as names, angles, and sides.
6. Circles—facts, center, radius, circumference, diameter, arc, sector, segment.
7. Equality, construction, and use of triangles of all types.
8. Construction problems—angle, triangle, square, rectangle, regular polygons, perpendicular bisector of a line, perpendicular to a line, line parallel to another line, circle in a triangle, circle around a triangle, polygon in a circle, polygon around a circle.
9. Problems dealing with areas of triangles, quadrilaterals, polygons, circles.
10. Problems dealing with tangents to a circle, chords of a circle.
11. Problems employing angles measured by degrees.

12. Problems of symmetry.
13. Problems employing ratio and proportion.
14. Problems employing graphical method of representing geometrical and other truths.
15. Problems involving measurement, dealing with lines or surfaces or solids, including volumes.

The method of teaching geometry depends upon drawing, paper-cutting, the use of instruments outdoors, and observation. To secure the best results a great many figures must be constructed neatly and accurately. In order to accomplish this result it has been found necessary to provide each pupil with the following instruments: a drawing-board, **T** square, two right triangles—one 45, the other 30-60—a ruler, compass, pencil, protractor, colored crayons, and scissors. The entire equipment of each pupil is not expensive and has been found to save the desks from damage due to the use of sharp-pointed compasses. The teacher has a pair of blackboard compasses, a straight-edge ruler, a yardstick, and a blackboard protractor. Plain paper, ruled paper, and squared paper have been found necessary, although but little of the squared paper is needed. Most of the drawings are made upon plain paper, some of it arithmetic paper, some of a better grade. All the drawings are kept in an envelope to prevent loss and soiling.

The children learn the use of the drawing-board, **T** square, and one right triangle, preferably the 30-60, by drawing to scale the floor plan of their schoolroom and similar rectilinear figures. Actual measurements are made of the schoolroom; the class determines the scale to be used and then receives instructions on how to use the simple instruments. Similar problems are suggested by the teacher, such as drawing to scale the window, top of the desk, blackboard, comparing the window space with the floor space to see if the room is properly lighted; then additional problems are requested from the children, such as designs for brick walks, checker boards, concrete sidewalks,

and other pieces of construction which employ rectangles. Enough of these figures are executed on the drawing-board to make children perfectly familiar with the use of the T square and the triangle. It has been found necessary to give a large number of problems of this simple kind so that the class will surely understand how to handle these instruments. Each of the other instruments is taught in succession, and a sufficient number of problems involving the use of each instrument is introduced to make sure that the pupils can handle each instrument perfectly. From the beginning the class works from directions written upon the board. At first the teacher gives all of these directions, stating carefully what the pupils are to do, what instruments they will employ, and what scale they will use. After the novelty has passed, the children gradually begin to make their own directions for solving a given problem and also determine their own scales. These directions are written upon the board, and the class solves this problem according to its own directions. None of these directions is written upon the paper on which the drawings are made; the only statement appearing there is the statement of the problem, and accompanying it the drawing which solves the problem.

Closely related to this method of developing power to analyze geometrical problems is the optional outside work, which includes observation and invention of problems. Such observation on the part of pupils at home, on the street, in public and private buildings, leads naturally to pupils' original problems for class solution. The brighter pupils are first encouraged to bring in such problems based upon their observation, giving full directions for their solution, and after the problems are worked out by other members of the class, they are expected to show their solution of the problem. Pupils of this grade imitate very easily and very quickly, and a large number of drawings of figures which are not studied regularly in the class period are posted about the room. These give opportunity

for the brighter pupils to do more work in analyzing figures, determining the scale used, and reproducing these figures or making improvements upon them according to their own ideas. Such optional work inside the classroom makes it possible for the teacher to devote considerable time to those who need individual attention at the same time that it provides plenty of work for the more able part of the class. The results from this method are extremely worth while.

Problems involving the use of the compasses are introduced into the course by means of simple work in making semicircles and arcs of circles of all kinds as applied in ornamental design. At first these designs make use of circles only. Gradually other figures are introduced with the circles, so that triangles, squares, and other polygons form the basis of a design which is constructed by means of T square, triangles, and compasses, and the work at the same time is properly motivated.

Problems which make use of the protractor are next introduced. They involve the measurement of angles of various kinds which may be drawn at the direction of the teacher, or the angles upon any given map, of the city or of the state or of the country, may be measured and results compared. Problems in construction work, depending upon the protractor as an instrument, are then taught by construction of figures such as stars of various numbers of points, angles equal to given angles, and problems of like nature, so that the pupils become thoroughly familiar with this use of the protractor. The protractor also comes into play in a great deal of the surveying which is done outdoors.

Certain important things are demanded of each pupil from the start: accuracy, neatness, spacing upon the page, and good lines. These demands can be met only by the use of the simple instruments which have been named. In no other way could a large number of figures of all sorts be drawn accurately and neatly.

The class is taught in the beginning somewhat as follows: The teacher first tells the class members briefly what they are to do—that is, the problem for the day and in general how they will accomplish the solution of the problem. Emphasis is laid upon the necessity of understanding what they are to do before they do anything. Clear, full, and simple directions, placed upon the blackboard, are given at first. The teacher then draws upon the blackboard a figure which illustrates the problem under discussion and employs the instrument with which the pupils are to become familiar. Then the class constructs the figure, while the teacher works with individual pupils to secure carefulness and accuracy in the construction work. There are so many apparently unimportant things which must be done by each pupil in order to secure the best results that from the start the teacher is under the necessity of watching each member of the class to see that these little things are properly attended to. Some of these are: keeping the T square close to the left edge of the drawing-board, holding the compasses properly, placing the protractor at the right point for measuring or constructing angles. In brief, the little details which make the difference between accuracy and inaccuracy must be carefully taught, and the children must be so habituated to them that early in the course they invariably make good figures. The importance of this is realized when it is understood that children see things exactly as they are. It is impossible for them to understand that any angle, line, or figure may be considered as anything other than it is. None of the suppositions possible in plan geometry of the demonstration type is at all possible in concrete geometry. Therefore the figure illustrating the problem under discussion must be drawn with as much exactness and nicety as is within the power of the children to accomplish.

As soon as the instruments have been carefully taught and can be handled with ease, the members of the class are divided

into groups of not over six, each with a captain, and these groups are sent about town to observe various forms of geometry as seen upon public and private buildings in the course of the outdoor trip. Each party must make careful sketches in a small notebook, estimate measurements, and secure sufficient data about the designs and other applications of geometry which they have seen to be able during subsequent periods to make accurate drawings in the schoolroom to scale. Such outside observation, carefully organized, with its purpose understood before the children undertake it, produces excellent results. In the first place, better designs are made upon the drawing-board, and a better appreciation is felt for the everyday occurrence of geometrical forms. Another result from these trips is the presentation of new problems involving considerable thought in their solution. The question arises: How shall we draw accurately to scale the design of a window, a door, an ornamental gateway, the arch upon a belfry, and similar problems?

Another form of outdoor trip which is used to teach the value of angles, accuracy in measurement of angles, boundaries, and the like makes use of the simple tripod transit or plane table, such as is described in various boys' books having to do with elementary surveying. This transit makes use of a protractor for measuring angles, and by its use the pupils are taught the vital need of accuracy in laying out angles in city lots, city blocks, and wherever land measurements are concerned. These simple field surveys are carried out in the same way as the observation trips mentioned before. Each party has a captain, and each one in the party has an opportunity to measure angles and to use the fifty-foot tape to get distances. Each pupil in a party makes an accurate drawing to scale from the field notes thus taken.

A strictly logical method of presenting the subject-matter is not found necessary, but rather the reverse is true. A great

many geometrical forms are taught in their simplest forms and are brought up again and again in combination with other forms; hence the ability to handle those previously learned is made use of continually. Thus the proper gradation of subject-matter is secured. The application of geometry to all sorts of designs for interior decoration, floors, linoleums, wallpaper, and the like, as well as the same applications in architecture, furnishes the basis of this constant review of figures already learned. The value of such material lies chiefly in the fact that it broadens the appreciation of the pupil for the applications of geometry. It also causes a great deal of well-motivated work in mechanical execution.

Every possible source is used which supplies material of any sort which is likely to come within the experience of the child. This includes the study of maps to learn the value of accurate surveys—furnishing another reason for exactness in drawing angles, whether in laying out small lots of land or immense tracts comprising the area of an entire state.

Drawing to scale is a large and necessary part of all the work. Every outdoor problem must be reduced to a scale drawing. Every indoor problem likewise and more than one-third of the entire course make use of this method of representing figures of all sorts upon paper. This extended use of drawing to scale is extremely valuable because very little of such work has been done, and the training in the use of drawing to scale and the accurate reading of drawings to scale form a necessary part of any person's education. This work leads naturally to the consideration of proportion, a subject which is greatly neglected in the study of mathematics. Proportion is used in dealing with similar triangles, and these form the basis of considerable study in outdoor measurements of heights and distances. The ability to solve simple problems in proportion grows naturally out of the work with similar triangles and is within the grasp of the seventh-grade child.

The value to any pupil of the knowledge required in solving simple equations of the proportion type is not to be overestimated. There are so many instruments used to determine distances which employ similar triangles that it is very easy to interest children in this subject, and by that means to secure for them a mastery of it at a sufficiently early stage in their development to give proportion no terrors for them. Both in algebra and in geometry of the traditional type, proportion has held a place altogether too insignificant. Rugg and Clark ask, "Is there anything more fundamental to give a student in high-school mathematics than a clear grasp of the vital principle of variation, of proportionality, of functionality?"

A group of pupils requires for outdoor use quite a number of simple instruments used in measuring heights and distances and laying off and measuring angles. All these instruments are made in the class in manual training. They include the tripod transit mentioned above, wooden angles of various kinds, stakes, rods, and similar articles necessary for carrying out the simple but accurate survey of small lots of land. This kind of manual-training project is always extremely well done because there is a motive behind its construction. Similarly, whatever color schemes are introduced in the course of reproducing designs or inventing new designs are secured from the drawing department. In this way the children learn the interrelationship between correct color and correct design so that they understand them to go hand in hand. The extent to which this phase of geometry can be carried depends entirely upon the teacher. Design, color scheme, and application find their place in manual training, in drawing, and in sewing. As illustrations of this statement: checker boards are made in the manual-training class from designs furnished by the concrete-geometry class with color schemes secured from the drawing department; calendars are designed mechanically in the concrete-geometry class and colored in the drawing class; de-

signs originate in the geometry class for use on a simple table runner, the color scheme coming from the drawing department, and the embroidering being done in the sewing department; a design for making a curtain with stick printing originates in the geometry class, but the sticks are made in the manual-training department and the coloring of the material is done in the drawing class, where the color scheme originated.

The tripod transit may be made in the class in manual training, or a substantial wooden tripod may be purchased from any dealer in photographic supplies. A plumb bob can be secured from a local hardware store and attached to the underside of the tripod head. Then a piece of board approximately eleven inches by fourteen inches is fastened upon the tripod head so that the center of the board is over the center of the tripod head. A small pin is set into the center of this larger board. If this tripod transit is set up level at the vertex of an angle to be measured in the field, the plumb bob should be placed at the vertex of this angle.

This transit may be varied in a number of ways. It may be used with pins only, and the angle may be found by sticking pins into the board. Another way of securing the angles is by the use of the semicircular protractor placed upon the board with its center against the pin. Still a third way is to secure a circular protractor, fasten it securely upon the board, correctly centered, and then place a wooden pointer upon this circular protractor with one end at the center. Any of these methods will give good results.

A sighting-stick consists of a rod at least six feet long, with a short piece screwed to this rod so that it can be adjusted at any given angle. This is used for quickly estimating distances and heights where it is important to give these measurements or to teach the possibility of getting such measurements without the use of more expensive instruments. In the same way triangles of wood of the equilateral and

isosceles types are constructed and are used in the same manner in the field for the same purpose.

It is to be understood that none of these instruments is original; they have been in use for centuries, and some of them are in use even down to the present time in some form, but their use actually to study geometry in its first-hand applications, to appreciate its significance, has never been attempted to any extent in the lower grades of the secondary school. This is the phase of concrete geometry as taught in the seventh grade which is significant, because it teaches children that the world depends upon geometry as applied in all of these phases which they have studied outdoors. A prominent place is given in the course to the construction of various forms of graphs. These figures are introduced as soon as cross-section paper has been used to any extent, and the class soon learns how to make graphs of its own ability in the different daily subjects, besides making graphs of attendance, variations in temperature in connection with elementary science, and a few important facts for use in history classes. There is no attempt made to solve equations by the graphical method, for the familiarity which the pupils have in the seventh grade with graphical representation makes it possible to use the graph successfully in the algebra of the following year. In addition to the construction of graphs, considerable practice is had in reading and interpreting graphs of all kinds.

There is some work done in experimenting with various figures. For example, the rigidity of the triangle is tested by forming a triangle of three boards nailed together. The strength of this arrangement is compared with that secured by nailing four boards together to form a parallelogram. The triangle is also compared with other forms experimentally, and its importance in construction is definitely brought out. Examples of the use of those different types of construction and comparison between the types to determine which is the most

serviceable and which furnishes the greatest strength are secured by trips of observation. Bridge construction, trusses of all kinds, supports for telephone cross-arms, and similar uses of rods to form triangles teach the class the importance of this geometrical form. By experiment and by observation the value of different forms is thoroughly understood.

Design holds a large place in the course and consists of a series of problems which deal with useful and ornamental articles which boys and girls can easily make. In the beginning simple designs are taught and variations of them are suggested. A few simple color schemes are introduced. Home study of design is encouraged, and pupils are urged to bring to the class sketches of such designs as they find in their homes which have not been studied in class. The girls are urged particularly to design such things as can be useful in the home, such as table runners, pillow covers, curtains, and other ornamental and useful articles of like nature. The boys are urged to design articles which they can make or to furnish designs of articles which the girls can make. The use of all of the mechanical instruments is brought into play, and the proper spacing of different figures in geometry is studied by comparison with good designs seen at home and in the schoolroom. Criticism of the designs furnished by different pupils takes place in the class and is secured by comparison with good design as furnished in standard designs found in all sorts of wall-papers, linoleums, curtain patterns, and things of like nature.

A section of geometry which furnishes a constant, varied, and well-motivated quantity of material is the study of arches. The study of arches from the simplest to the most complicated type begins in the outdoor observation trips, when sketches are made freehand of arches as found upon various buildings observed by the class, and the method of construction of the various types is then studied in the class. These figures, which apparently are very simple in construction, furnish a great

deal of work in bisection of lines, location of centers of arcs, formation of correct angles for the bases of arcs, and in general train the children to understand what lies behind the structure of these simple curves. Once more the importance of the triangle is brought to the attention of the class because so many arches are formed upon the triangle as a basis. If the locality is not favorable for the study of arches as found in public and private buildings, pictures of the famous cathedrals and other public and private buildings of the world furnish a wealth of material for the study of this type of geometry.

Rose windows, such as are found in every cathedral, furnish another excellent series of problems for study because of the variety of operations necessary before the design is completed. Here again pictures may be made the basis of such study, provided local conditions do not present any examples. While this subject has to do almost entirely with the circle, there enters into the construction of rose-window designs a great many of the fundamental truths of geometry, the chief among these being angles, bisection of lines, concentric circles, and other similar fundamental truths. In such work the immense value of some truths which have not been apparent previously is fully brought out. Here again color schemes are needed, and at one more point is drawing correlated with mathematics.

The content outlined in this article is very evidently such as every well-educated member of society feels is a necessity. Every item of subject-matter is tested in this way, and its place in the course determined by this standard as a measure. The appreciation of environment is one of the definite results of this course, as is shown by the fact that many of the pupils say, "I see geometry everywhere." Probably the greatest criticism that has been made of the traditional course in plane geometry in the secondary school has been that it had no functional bearing in the education of the child. This is proved by the fact that many teachers studying the course in the teaching

of concrete geometry in the seventh grade have remarked that they never had previously appreciated the applications of geometry in everyday life. It is evident, then, that this kind of subject-matter is peculiarly fitted to a course in the junior high school, and, if carefully taught to a large number of seventh-grade pupils, will have an important effect upon the succeeding generations, because it supplies an element in education which has hitherto been greatly lacking.

A summary of the results obtained is interesting. One result of a general nature has been mentioned: that is, the appreciation of the universal prevalence of geometry in the environment. Another result equally valuable is that boys and girls acquire the ability to draw accurately from a set of directions simple figures in various ways. Still another result is increased power to analyze a set of directions and work quickly and accurately from these directions. Finally, the greatest result, that of a broader conception of space relations, is secured. Every problem and every method are focused upon that as an ultimate aim, and results show that this method of teaching concrete geometry in the seventh grade is meeting with reasonable success.